Modified descend curvature based fixed form fuzzy optimal control of nonlinear dynamical systems

Yusuf Oysal,∗ Yasar Becerikli, A. Ferit Konar

Department of Computer Engineering, Anadolu University, Eskisehir, Turkey
Department of Computer Engineering, Kocaeli University, Kocaeli, Turkey
Department of Computer Engineering, Dogus University, Istanbul, Turkey

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Abstract

In this study, a fuzzy rule-based optimal controller is designed for nonlinear dynamical systems. The direct second order method (or direct-descend-curvature algorithm) with a modification called ‘modified descend controller (MDC)’ is used for calculating the parameters of the fuzzy feedback controller. The optimal control problem defined here has dynamic constraints of nonlinear system states and static constraint of a known form of fuzzy controller. The form used here is a standard fuzzy system that uses singleton fuzzifier, product inference engine, center average defuzzifier, and with the Gaussian membership functions of the system states to be controlled. The design is developed by minimizing a quadratic performance index selected for the desired operating conditions. Successful simulation results of controlling the temperature of a continuous stirred tank reactor (CSTR) and a bioreactor are given.

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1. Introduction

Two main advantages of fuzzy systems for the control and modeling applications are (i) fuzzy systems are useful for uncertain or approximate reasoning, especially for the system with a mathematical model that is difficult to derive and (ii) fuzzy logic allows decision making with the estimated values under incomplete or uncertain information (Zadeh, 1975). For static systems, especially for function approximation, there are lots of methods to obtain fuzzy system parameters such in Jang (1993), Wang (1997) and Wang and Mendel (1992a,b). But determination of the fuzzy system parameters of the dynamical systems for control and modeling applications is not always easy due to the complexity and nonlinearity. This study proposes to design fuzzy controllers for nonlinear dynamical systems using optimal control algorithms. So the term ‘fuzzy optimal control (FOC)’ comes from this idea (Wang, 1998). There are many applications of optimal control for fuzzy controller designs of dynamical systems. In Heckenthaler and Engell (1994), a fuzzy controller was designed for a two-tank system, which is a strongly nonlinear plant due to the characteristics of the valves. The rules of the fuzzy controller designs were derived from the approximate time-optimal control law. A number of stable and optimal fuzzy controllers were developed for linear systems by using the Pontryagin minimum principle with quadratic cost function and the application of the optimal fuzzy controller to the ball-and-beam system were presented in Wang (1998). Chen, Tseng, and Uang (1999) introduced a fuzzy linear control design method for nonlinear systems with a fuzzy linear model that provides rough control to approximate the nonlinear control system, and an optimal H-infinity scheme that provides precise control to achieve the optimal robustness performance. Moreover, Wu and Lin (2000a,b) presented local and global approaches of optimal and stable fuzzy controller design methods for both continuous- and discrete-time fuzzy systems under both finite and infinite horizons by applying traditional linear optimal control theory. Another approach, intelligent optimal control problem is
Nomenclature

Φ weighted function of finite time
θ(t) time-varying fuzzy system output membership function center parameters
σ_0 spread parameter of Gaussian membership function
b(t) time-varying local gain vector
c_0 center parameter of Gaussian membership function
CSTR continuous stirred tank reactor
d dimensionless deviation variable for the feed temperature disturbance of CSTR
De Damköhler number
DFN dynamic fuzzy network
FOC fuzzy optimal control
ITAE integral time multiplied absolute value of error
J performance index
k iteration number
K(t) time-varying gain matrix
L weighted function of system states and control vector
MDC modified descent controller
P Riccati matrix
PI proportional-integral
q Riccati vector
r rule number
SIMO single-input-multi-output
τ_i initial time
τ_f final time
TPBVP two point boundary value problem
u control signal (jacket temperature of CSTR, incoming flow rate for bioreactor)
W constraint matrix
x_1 dimensionless reactant conversion for CSTR, number of cells for bioreactor
x_2 dimensionless reactant temperature for CSTR, the amount of nutrients for bioreactor

Greek symbols

φ weighted function of finite time
σ_0 spread parameter of Gaussian membership function
θ(t) time-varying fuzzy system output membership function center parameters

considered as a nonlinear optimization with dynamic equality constraints, and dynamic fuzzy network (DFN) and dynamic neural network (DNN) as a control trajectory priming system. The resulting algorithm operates as an auto-trainer for DNN (a self-learning structure) and generates optimal feed-forward control trajectories in a significantly smaller number of iterations (Becerikli, 1998; Becerikli, Konar, & Samad, 2003; Becerikli, Oysal, & Konar, 2004).

Our study differs from these studies for the usage of second order gradient information of an optimal control method for fuzzy controller designs of nonlinear dynamical systems. The direct second order method (or direct-descend-curvature algorithm) that is one of the best algorithms of optimal control is used for calculating the parameters of the fuzzy feedback controller with a modification called as modified descent controller (MDC) algorithm. The optimal control problem defined here has dynamic constraints of nonlinear system states and static constraint of a known form of fuzzy controller.

The organization of this paper is as follows. In Section 2, the description of a standard fuzzy system will be given. Section 3, the optimal fuzzy control problem will be introduced. The solution algorithm of this problem will be given in Section 4. In Section 5, some of the useful formula for a quadratic performance index is derived. Finally, in Section 6, simulation results for a CSTR and a bioreactor will be presented.

2. Standard fuzzy system

A fuzzy system consists of linguistic IF-THEN rules that have fuzzy antecedent and consequent parts. It is a static nonlinear mapping from the input space to the output space. The inputs and outputs are crisp real numbers and not fuzzy sets. The fuzzification block converts the crisp inputs to fuzzy sets, and then the inference mechanism uses the fuzzy rules in the rule-base to produce fuzzy conclusions or fuzzy aggregations, and finally the defuzzification block converts these fuzzy conclusions into the crisp outputs. We call the fuzzy system with singleton fuzzifier, product inference engine, center average defuzzifier and Gaussian membership functions as “standard fuzzy systems” (Wang, 1997, 1998). Nonlinear mapping of a standard fuzzy system that has n state variables is in the following form:

\[ f(x) = \frac{\sum_{i=1}^{r} \theta_i \prod_{j=1}^{n} \exp(-1/2)((x_j - c_{ij})/\sigma_{ij})^2}{\sum_{i=1}^{r} \prod_{j=1}^{n} \exp(-1/2)((x_j - c_{ij})/\sigma_{ij})^2} \]  

where \( r \) is the number of rules, \( c_{ij} \) and \( \sigma_{ij} \) are the center and spread parameters of the \( i \)-th rule and \( j \)-th input variable of the corresponding Gaussian membership function, respectively. \( \theta_i \) is the center parameter of the \( i \)-th rule output membership function. For an \( n \)-dimensional dynamical system with single-input and multi-output (SIMO), a standard fuzzy controller can be defined with the following equation:

\[ u(t) = \frac{\sum_{i=1}^{r} \theta_i(t) \prod_{j=1}^{n} \exp(-1/2)((x_j(t) - c_{ij})/\sigma_{ij})^2}{\sum_{i=1}^{r} \prod_{j=1}^{n} \exp(-1/2)((x_j(t) - c_{ij})/\sigma_{ij})^2} \]  

Here \( x(t) \in \mathbb{R}^n \) represents the state of the system to be controlled. The minus sign in the control term comes from the feedback control law in linear dynamical systems that is \( u = -Kx \), where \( K \) is an \( n \times n \) gain matrix calculated from the solution of Riccati differential equations of linear quadratic regulator problem.

Defining the time varying parameter and base function vectors as:

\[ \theta(t) = [\theta_1(t) \cdots \theta_r(t)]^T \]  

\[ \xi(x(t)) = [\xi_1(x(t)) \cdots \xi_r(x(t))]^T \]  

\[ \dot{\xi}(x(t)) = \frac{\partial \xi(x(t))}{\partial x} \]
where each components of the base function vectors can be expressed as:
\[ q_k(x(t)) = \prod_{i=1}^{r} \exp\left(-\frac{1}{2}(x_i(t) - c_{ij})/\sigma_{ij}\right)^2 \]
\[ k = 1, \ldots, r \]

The feedback controller for a SIMO system given in (2) can be considered as the general form of the linear feedback law that can be rewritten as:
\[ u(x(t), t) = -\phi(t)x(t) \quad \text{or} \quad u(x(t), t) = -L^T(x(t)) \theta(t) \]

The only difference between them is that dynamic system states are measured and feedback in linear controller whereas in fuzzy controller calculation of base function values of system states are measured and feedback in linear controller whereas in fuzzy controller calculation of base function values of system states is also needed. In general form block diagram of the optimal fuzzy controller design is shown in Fig. 1 in which \( \phi \) is the desired operating state of this system.

3. Fuzzy optimal control (FOC) problem

In a traditional optimal control problem such as a tracking problem, a performance criteria or index is selected such that whenever it is minimized, the states of the system of our interest will track the desired trajectories. In other words, the cost or the penalty of the performance index will be minimum around the desired trajectory demands. In this study, an optimal control form known as Bolza problem (Lewis, 1992) is selected with the following performance index:
\[ J = \phi(x(t)) + \int_{t_0}^{t_f} L(x(t), u(t)) \, dt \]

where \( J \) is a scalar function of the dynamical system states \( x(t) \) and control \( u(t) \), \( t_0 \) is the initial time and \( t_f \) is the final time of the control action, \( \phi \) is the weighted function of the final time states \( x(t_f) \) and \( L \) is the weighted function. The dynamical system with fuzzy feedback controller can be in general represented with the following differential equation:
\[ x(t) = f(x(t), u(t), t), \quad x(t_0) = x_0 \]

From these equations and substituting the fuzzy feedback control law given by (6) in these equations, an optimal fuzzy control problem with a known form of the controller (that is fixed form optimal control) can be defined as:
\[ \text{Minimize} \quad J = \phi(x(t)) + \int_{t_0}^{t_f} L(x(t), \theta(t)) \, dt \]
\[ \text{Subject to:} \quad x(t) = f(x(t), \theta(t), t), \quad x(t_0) = x_0 \]

4. Solution of the FOC problem

For solving the FOC problem or for calculating the time-varying fuzzy system parameter vector \( \theta(t) \), a second order direct performance index minimization method can be used (Mitter, 1966). This method is based on the Taylor series expansion that uses similar solution techniques as Newton. The solution steps of the known form FOC problem with the second order direct minimization method can be summarized as follows (Oysal, 2002), see Appendix A:

1. Select the initial values of the stopping criteria parameters \( W^0, y, \gamma \) where \( W^0 \) is the initial constraint matrix (W) of fuzzy system parameter values. Also select the initial fuzzy system output parameters \( \theta^0(t) \).
2. Select the number of rules \( r \) arbitrarily until a satisfactory results are obtained or select by defining the fuzzy sets to cover the input and output spaces from some expert’s knowledge, and set iteration number \( k \) as zero.
3. Determine the center \( c_j \) and spread \( \sigma_{ij} \) parameters of Gaussian membership functions of input variables (dynamical system states) of the fuzzy controller by considering the possible maximum and minimum values.
4. For \( k = 0 \), calculate the first control input value \( u_0 = -(\hat{\theta}^0)^T(t)(\hat{x}^{k-1}) \) (set \( \hat{x}^{k-1} = x(t_0) \))
5. Calculate \( \hat{x}(t) \) for \( t_0, t_1 \) by solving \( \hat{x} = f(x, u) \), \( x(t_0) = x_0 \).
Evaluate the time-varying matrix function values $A(t)$, $B(t)$ and $C(t)$, and vector function values $x(t)$ and $w(t)$ from Equations (A.12)-(A.16).

(6) Determine the final values $\phi_1(t)$ and $\phi_2(t)$ of the Riccati equations.

(7) Calculate symmetric Riccati matrix $P^i$ and $q^i$ vectors by backward integration of the Riccati differential equations from Equations (A.10) and (A.11).

(8) Calculate the gain matrix $K^i(t)$ and local gain vector $b^i(t)$ form Equations (A.8) and (A.9), respectively.

(9) Calculate the new state values of the dynamical system from

$$ x = f(x, u^i + b^i(t)) = f(x, u^i - \hat{\theta}^T(x^{-1})y(K^i(t))$$

$$ \times [x - x^i] + b^i(t)), \quad x(t) = x_0 $$

(10) Calculate the incremental values

$$ \begin{align*}
\hat{\theta}^i &= \theta^i - \hat{\theta}^i = \theta^i - \hat{\theta}^i \\
\hat{\theta}^i &= \theta^i = \theta^i
\end{align*} $$

(11) Evaluate $\hat{x}^i$ from $\hat{\theta}^i + b^i(t)$ and new values of the fuzzy system output parameter values can be calculated from $\theta^{\ast}(t) = \theta^i + \hat{\theta}^i(t)$.

(12) If $\|\hat{H}_0\| > \gamma$ and $W \gamma < \gamma W$, then stop and print the output values: $\theta^{\ast}(t) = \theta^{\ast}(t)$, $x^i(t) = x^{\ast}(t)$, $K^i(t) = K^i(t)$ and $b^i(t) = b^i(t)$.

4.1. Performance criteria analysis

In this paragraph, necessary formulas for the solution of the FOC algorithm for a SIMO dynamical system will be derived. The performance criterion is selected as in the quadratic form given by:

$$ J = \frac{1}{2}(x(t) - x_d(t))^T S(x(t) - x_d(t)) $$

$$ + \frac{1}{2} \int_{t_0}^{t_f} \left[ (x(t) - x_d(t))^T Q(x(t) - x_d(t)) ight. $$

$$ \left. + w(t)^T R w(t) \right] \, dt $$

where $x_d(t)$ is the desired operating states, and $S$, $Q$ and $R$ are suitably chosen positive-semidefinite or definite symmetric matrices. If we select the control action given by (6), then the quadratic index dependent only on the system states:

$$ J = \frac{1}{2} \int_{t_0}^{t_f} \left( x(t) - x_d(t) \right)^T S \left( x(t) - x_d(t) \right) $$

$$ + \frac{1}{2} \int_{t_0}^{t_f} \left[ \left( x(t) - x_d(t) \right)^T Q \left( x(t) - x_d(t) \right) ight. $$

$$ \left. + \hat{\theta}^T(t) \Omega \left( x(t) \right) \hat{\theta}^T(t) \right] \, dt $$

The derived equations necessary for the calculation of the matrices or vectors of Riccati equation coefficients and the final values of the Riccati variables are listed as below (Oysal, 2002):

$$ f_i = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} $$

$$ f_\theta = \begin{bmatrix} \frac{\partial f_1}{\partial \theta} & \cdots & \frac{\partial f_1}{\partial \theta} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial \theta} & \cdots & \frac{\partial f_n}{\partial \theta} \end{bmatrix} $$
5.1. CSTR temperature control

Our first simulation of FOC algorithm is the temperature control of a simplified nonlinear CSTR. The CSTR model was taken from Ray (1981). The relevant equations are given below:

\[
\frac{dx_1}{dt} = -x_1 + \frac{Da(1-x_1)e^{x_1/(1+x_2/y)}}{T_0} 
\]  
\[
\frac{dx_2}{dt} = -x_1 + BDa(1-x_1)e^{x_1/(1+x_2/y)} - \beta(x_2 - x_2) + \frac{f0}{\gamma} 
\]  

The first of these is the dimensionless reactor component continuity equation, the second the dimensionless reactor energy equation. A complete explanation of the model is given in the cited reference. Here we note that \(x_1, x_2\) represent dimensionless reactant conversion and temperature defined as:

\[
x_1 = \frac{C_{A1} - C_A}{C_{A1}} \quad (31) 
\]

\[
x_2 = \frac{T - T_0}{T_0} \left( \frac{E}{RT_0} \right) \quad (32) 
\]

where \(T\) and \(C_A\) are the reactor temperature and concentration, respectively. \(C_{A1}\) is the inlet reactant concentration and \(T_0\) is the nominal design value for reactor temperature. \(Da\) is the Damköhler number that may vary as the reactor catalyst temperature, reactant conversion and temperature changes and the variables \(d, u\) are dimensionless deviation variables for the feed temperature disturbance and control (jacket temperature), respectively. In the simulations two nominal parameter values are considered. In the second case the sensitivity of this system is too much. \(Da = 0.135, B = 11, \beta = 1.5\), and \(y = 20\).

To apply the fuzzy optimal control algorithm, it is necessary to calculate the gradients of CSTR system state functions, i.e., the right hand side of the equations given by (29) and (30) with respect to system states and fuzzy parameters. As a result we obtain:

\[
f_s = \begin{bmatrix} 
-1 + Da \exp \left( \frac{x_1}{1 + x_2/y} \right) 
-BDa \exp \left( \frac{x_1}{1 + x_2/y} \right) 
\end{bmatrix}
\]

\[
f_o = - \frac{\partial f_s}{\partial x} T
\]

5. Simulation results

Our design of FOC algorithm will be demonstrated with two examples of two-benchmark chemical process control. Although in this algorithm only output center parameters are calculated while other parameters of the fuzzy system are kept fixed, it is aimed to show the effectiveness of the FOC algorithm in the control of nonlinear dynamical systems. Our first set of experiments examine the fitness of this issue by utilizing a continuously stirred tank reactor (CSTR) process (Ray, 1981), and the second set of experiments used data generated from a bioreactor (Ungar, 1991).

5.1. CSTR temperature control

Our first simulation of FOC algorithm is the temperature control of a simplified nonlinear CSTR. The CSTR model was taken

\[
T(0) = 300, \quad R = 1.98 \quad (35)
\]
For the first experiment on CSTR temperature control nominal parameters are $B = 7.0$, $\beta = 0.5$, $Da = 0.11$. By considering the amount of possible changes in the dimensionless reactant conversion and temperature, for the step three of FOC algorithm, the membership functions of the input states of the feedback fuzzy controller is selected as in Fig. 2. All time varying output parameters $\theta(t)$ of the fuzzy controller are firstly selected as one. Fig. 3(b) shows the initial CSTR state changes and the input. The optimal trajectories obtained with the FOC algorithm are also shown in Fig. 3(b).

In addition, we can see the effectiveness of the FOC algorithm by looking at the changes of the coefficients of the Riccati matrix and the vector given in Fig. 4 in which the $P_i$ denote the $i$th row $j$th column element of the $P$ matrix and $q_k$ denote the $k$th row element of the $q$ vector calculated from (14) and (15). As we see these parameters are decreased nearly to zero from very larger values. Fig. 5 shows time-varying output parameters $\theta(t)$ of the fuzzy controller after the application of the FOC algorithm. When we examine this figure, it seems that these parameters can be partitioned into three different types of curves. This result means that three membership functions for output

![Fig. 2. Fuzzy system membership functions for the dimensionless reactant conversion ($x_1$) and temperature ($x_2$) ($B = 7.0$, $\beta = 0.5$, $Da = 0.11$).](image)

![Fig. 3. Dimensionless CSTR states and the control input.](image)

![Fig. 4. The initial and final (after FOC algorithm) time-varying Riccati matrix and vector parameters.](image)
Fig. 5. Optimal fuzzy system output parameters $\theta(t)$ obtained after the FOC for CSTR temperature control. For three membership functions for each state, then we have nine time-varying rules with the above output parameter curves.

In the second experiment on CSTR, nominal parameters are chosen as $B=11$, $\beta=1.5$, and $Da=0.135$. With these parameters, limit cycle oscillations exist for feed temperatures in some approximate ranges and small parameter changes can produce dramatic changes in the steady state and dynamic behavior of the CSTR in the open loop (Ray, 1981). That is the reason why we select this case to test the efficiency of the FOC algorithm. For the step three of FOC algorithm, five Gaussian membership functions are chosen for each state of CSTR shown in Fig. 6. This means that fuzzy system consists of 25 rules, i.e., 25 time-varying output parameters $\theta(t)$. Fig. 7 shows the changes in CSTR temperature and the control input for some iteration. For about eight iterations, the temperature converges to the desired value. Finally some of the optimal fuzzy system parameters obtained by the FOC algorithm can be seen in Fig. 8.

Another simulation that is done is a regulator problem for observing the closed loop behavior of CSTR with FOC. For this case, the desired nominal dimensionless operating temperature is $x_2 = 2$ ($T = 330$ K). For $302 \leq T_f \leq 307$ K feed temperature values in CSTR system, limit cycle oscillations can be seen (Ray, 1981). Thus a step increase of 5 K in $T_f$ at $T_f = 300$ K causes the reactor to oscillate for the open loop situation (Fig. 9(a)). To overcome these oscillations FOC algorithm is implemented with a local dynamic optimization problem given in Appendix A. Closed loop responses of CSTR with PI controller (Ray, 1981) and with fuzzy optimal controller can be seen in Fig. 9(b). For a change from $T_f = 300$ to 305 K the closed loop system behaves much better than the open loop case. So any local changes due to disturbance effects can be eliminated with local feedback gains of FOC design.

It can be seen also in simulation results that FOC give a successful result according to the integral time multiplied absolute
value of error (ITAE) with a value of 140.73 where with PI controller ITAE is 238.73.

5.2. Bioreactor cell control

The bioreactor considered here is a tank containing water, nutrients, and biological cells. Nutrients and cells are introduced into the tank and they are mixed in it. Continuous time equations of this plant dynamics are given by (Ungar, 1991):

\[
\frac{dx_1}{dt} = -x_1u + x_1(1 - x_2)e^{x_2/\gamma}
\]

\[
\frac{dx_2}{dt} = -x_2u + x_1(1 - x_2)e^{x_2/\gamma} \frac{(1 + \beta)}{(1 + \beta - x_2)}
\]

(36)

(37)

The number of cells \(x_1\) and the amount of nutrients \(x_2\) characterize the state of this process. The volume of the tank content is maintained at a constant level by removing tank contents at a rate equal to the incoming flow rate, which is denoted by \(u\). In the above mathematical model, \(\beta = 0.02\) is the growth rate parameter, \(\gamma = 0.48\) is the nutrient inhibition parameter. The bioreactor control problem is to keep the amount of cells at a desired level. The stable state of this process is defined to be \(x_1 = 0.1207, x_2 = 0.8801\) and \(u = 0.7500\) (Ungar, 1991). In our simulations it is desired to bring the number of cells from this stable state to \(x_1(t) = 0.15 - 0.0293 \cos(8\pi t/15)\) from initial time \(t_0 = 0\) to final time \(t_f = 50\). The reason for choosing a sinusoidal output structure here is to demonstrate the controlling capability of our fuzzy optimal controller design for servo problems in the case of abrupt changes in the set point signal. Concerning the limits of bioreactor state changes, three fuzzy system input membership functions are selected for each state (Fig. 10). For about 10 iterations, optimal fuzzy output parameters are obtained that gives the best result for the desired operation states (Fig. 11). All fuzzy system output parameters converge to \(-0.9\) before \(t = 10\), so only this part is shown in Fig. 12.

The fuzzy optimal control algorithm combined with modified descent controller algorithm for fixed form optimal control provides an excellent combination for nonlinear process control for servo and regulator problems. It is seen from the simulation

Fig. 8. Some of the optimal fuzzy parameters obtained by the FOC algorithm.

Fig. 9. (a) Open loop CSTR temperature and (b) closed loop CSTR temperature with FOC and PI controller to a 5 K increase in feed temperature (CSTR parameters: \(B = 11.0, \beta = 1.5, \gamma = 20, Da = 0.135\); PI parameters: \(K_p = 3.11, K_i = 0.01\)).

Fig. 10. Fuzzy system membership functions for the number of cells (\(x_1\)) and the amount of nutrients (\(x_2\)).
Fig. 11. Desired and actual bioreactor states.

Fig. 12. Optimal fuzzy system output parameters \( \theta(t) \) obtained after the FOC for bioreactor cell control.

results that, the proposed controllers cause less frequency drop and oscillations in frequency rapidly damp out. Secondly, the simulation results show that efficient computational models and algorithms can be designed for fuzzy optimal controller design for robust control of fully nonlinear dynamical systems in general and for specific applications.

6. Conclusions and future works

In this work, we developed an algorithm to find the parameters of a feedback fuzzy controller for nonlinear dynamical systems. We have used a second order (curvature) direct descent algorithm with a modification to generate optimal time-varying fuzzy system output parameters. This algorithm is fast and robustly convergent if the initially suitable parameters are chosen. Successful simulation results have been obtained for highly nonlinear chemical processes such as a CSTR (Ray, 1981) and a bioreactor (Ungar, 1991).

The form of the fuzzy controller used in this study is a standard fuzzy system where the input membership functions are selected with respect to the dynamic system state changes and kept fixed. It would be fruitful for further researches to investigate when using other types of fuzzy systems such as Takagi–Sugeno fuzzy logic controllers (Takagi & Sugeno, 1985) and allowing the change of other fuzzy system parameters with different membership functions.

Appendix A. Modified descent controller algorithm for fixed form optimal control problem

This method is based on the development of the Bolza problem defined by (9) and (10). Performance index is directly expanded to Taylor series up to second order term around arbitrary known trajectories \((s, \theta)\):

\[
J(x + \delta x, \theta + \delta \theta) = \phi(x(t_f)) + \phi^T(x(t_f)) \delta x(t_f) + \frac{1}{2} \delta x^T(t_f) \phi_{xx}(t_f) \delta x(t_f) + \int_0^{t_f} \left\{ L_s(x, \theta) \delta x + \frac{1}{2} \delta x^T \phi_{x \theta}(t) \delta \theta + \frac{1}{2} \delta x^T \phi_{\theta \theta}(t) \delta \theta + \frac{1}{2} \delta x^T \phi_{x \theta}(t) \delta \theta + \frac{1}{2} \delta x^T \phi_{\theta \theta}(t) \delta \theta \right\} dt \tag{A.1}
\]

where the subscripts indicate partial derivatives. The local performance index (LPI) then is:

\[
\Delta J = \phi^T(x(t_f)) \delta x(t_f) + \int_0^{t_f} \left\{ L_s(x, \theta) \delta x + \frac{1}{2} \delta x^T \phi_{x \theta}(t) \delta \theta + \frac{1}{2} \delta x^T \phi_{\theta \theta}(t) \delta \theta + \frac{1}{2} \delta x^T \phi_{x \theta}(t) \delta \theta + \frac{1}{2} \delta x^T \phi_{\theta \theta}(t) \delta \theta \right\} dt \tag{A.2}
\]
All terms are evaluated at $(x, t)$. To constrain the size of the fuzzy system output parameter vector function $\delta(t)$, we add a positive definite matrix $W$ to the $L_{\Omega \phi}$ term. Then, the final LPI becomes:

$$\Delta(t) = q(t) - \frac{1}{2} \Delta x^T(t) f(t) x(t) + \int_0^\infty \left( L_{\Omega \phi} x(t) + L_{\Omega \phi} x(t) \right) dt$$

The local dynamic model is also given by:

$$\delta x(t) = f(t) \delta x(t) + f(t) \delta \delta x(t) = 0$$

The global problem has now been converted to local dynamic optimization problem:

$$\min \Delta(t)$$

subject to $\delta x(t) = f(t) \delta x(t) + f(t) \delta \delta x(t) = 0$

And the resulting two-point-boundary-value-problem (TPBVP) is solved using the general Riccati transformation (Bryson & Ho, 1975; Bullock, 1966). The solution of this problem has been summarized as follows (Lapichus & Luus, 1966; Mitter, 1966):

$$d^d = \left( \frac{1}{2} \Delta x^T(t) f(t) x(t) + \int_0^\infty \left( \frac{1}{2} \Delta x^T(t) f(t) x(t) + \int_0^\infty \left( L_{\Omega \phi} x(t) + L_{\Omega \phi} x(t) \right) dt \right) \right)$$

where $k$ is the iteration number. The following assumptions are in force:

$$\phi_{\Omega \phi} \leq 0$$

positive semi-definite (A.18)

$L_{\Omega \phi} + W > 0$ positive definite (A.19)

$L_{\Omega \phi} x + L_{\Omega \phi} x + L_{\Omega \phi} x + L_{\Omega \phi} x \geq 0$ positive semi-definite (A.20)

The above assumptions are necessary to overcome the conjugate points during the Riccati differential equations solutions. The $W$ matrix plays an important role to force these assumptions. In the second order modified descent algorithm presented, $W$ matrix is updated at each iteration step as (Becerikli, 1998; Becerikli et al., 2003, 2004):

$$\theta^{k+1} = \theta^d$$

where

$$\theta^d = \left( \frac{1}{2} \Delta x^T(t) f(t) x(t) + \int_0^\infty \left( \frac{1}{2} \Delta x^T(t) f(t) x(t) + \int_0^\infty \left( L_{\Omega \phi} x(t) + L_{\Omega \phi} x(t) \right) dt \right) \right)$$

measures the convergence to the optimal solution. At convergence:

$$\lim_{k \to \infty} a_k = 0$$

References


